Status of Open Problems in the Thesis "Bounded Width Graph Classes in Parameterized Algorithms"

Lars Jaffke[1]

[1]University of Bergen, Norway
lars.jaffke@uib.no

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Abstract

In this document I keep track of the status of the open problems I mentioned in my thesis. If you are aware of any results I have missed or solved one of the problems, I would appreciate it if you got in touch with me.

1 The List

#	Description	Status	Comments
3.1	Is there an algorithm for TREE DECOMPOSITION running in time $2^{o(k^3)} \cdot n^{\mathcal{O}(1)}$?	solved	Yes. (E)
3.2	Is there some function $f\colon \mathbb{N} \to \mathbb{N}$ and an algorithm that given a graph G and an integer k, either decides that $\mathsf{mimw}(G) > k$, or outputs a branch decomposition of G of mim-width at most $f(k)$, and runs in XP time parameterized by k?	open	
3.3	Is there some constant c and an algorithm that given a graph G, either decides that $\mathsf{mimw}(G) > 1$, or outputs a branch decopmosition of G of mim-width at most c, and runs in polynomial time?	open	
3.4	Is there some fragment of MSO/First Order Logic of graphs, containing a problem that is not locally checkable, and whose corresponding MODEL CHECKING problem is solvable in time $n^{f(\lvert\phi\rvert, \mathsf{mimw})}$, if a branch decomposition of mim-width mimw of the input graph is provided?	solved	A&C DN (B)
4.1	For constant $q \geq 8$, is the clique-width of $(q, q-2)$-graphs bounded by a constant or not?	open	
4.2	Is there a polynomial-time algorithm for MAXIMUM INDEPENDENT SET when the input graph is given together with one of its branch decompositions of sim-width 1?	open	

9.1	Characterize the graph class Linear Mim-Width 1.	open	
9.2	Are all connected, acyclic (co-) (σ, ρ)-problems parameterized by the mim-width of a given (linear) branch decomposition of the input graph W[1]-hard?	open	(C)
9.3	Would an $n^{o(w)}$-time algorithm for some (σ, ρ)-problem parameterized by the mim-width w of a given (linear) branch decomposition of the input graph refute ETH?	solved	Yes. (F)
10.1	Is Fall Coloring parameterized by clique-width W[1]-hard?	solved	Yes. (A)
10.2	Would an $n^{2^{o(w)}}$-time algorithm for Fall Coloring, where w denotes the clique-width of the input graph, refute ETH?	solved	Yes. (A)
10.3	Is Clique Coloring parameterized by clique-width W[1]-hard?	open	
10.4	Is there a computable function $g\colon \mathbb{N} \to \mathbb{N}$ such that each graph of clique-width cw can be clique colored with at most $g(\mathsf{cw})$ many colors?	open	
10.5	Is there an $n^{2^{2^{o(\mathsf{cw})}}}$-time algorithm for Clique Coloring, where cw denotes the clique-width of the input graph, or would such an algorithm refute ETH?	open	
10.6	Is there a $2^{2^{2^{o(\mathsf{cw})}}} \cdot n^{\mathcal{O}(1)}$-time algorithm for 2-Clique Coloring or would such an algorithm refute ETH?	open	
10.7	Is b-Coloring NP-complete on circular arc graphs, or, more generally, on any graph class of constant mim-width?	solved	NP-c on $\bigcup$circular arc
10.8	Is b-Coloring parameterized by the mim-width of a given branch decomposition of the input graph plus the number of colors XP?	solved	Yes. (B)
10.9	For which function $f\colon \mathbb{N} \to \mathbb{N}$ does it hold that for all fixed $k \geq 3$, b-Coloring on graphs of clique-width cw can be solved in time $\mathcal{O}^\star(f(k)^{\mathsf{cw}})$ while an algorithm running in time $\mathcal{O}^\star((f(k) - \epsilon)^{\mathsf{cw}})$, for any $\epsilon > 0$, would refute SETH? What about Fall Coloring, or Clique Coloring?	open	
10.10 (I)	Is b-Coloring parameterized by the treewidth of the input graph W[1]-hard?	solved	Yes. (D)
10.10 (II)	What is the fastest algorithm for b-Coloring parameterized by treewidth under the ETH?	open	

(A). J., Lima, Lokshtanov [STACS 2021].

(B). Bergougnoux, Dreier, J. [arXiv:2202.13335, SODA 2023].

(C). Bakkane, J. [IPEC 2022]: Dichotomies for minimization problems, and for maximization problems when σ and ρ are finite.

(D). J., Lima, Sharma [arXiv:2209.07772]: b-Coloring parameterized by pathwidth is XNLP-complete, which implies it is W[t]-hard for all t.

(E). Korhonen and Lokshtanov [arXiv:2211.07154]: A $2^{\mathcal{O}(k^2)}n^4$ time algorithm that given a graph G decides if G has a tree decomposition of width at most k.

(F). Bergougnoux, personal communication 2022: Independent Set given a linear branch decomposition of mim-width w cannot be solved in $n^{o(w)}$ time unless the ETH fails.